SHRI VISHWAKARMA SKIL UNIVERSITY

(CLASS NOTES)

SUBJECT: ZDSC-103(APPLIED MATHEMATICS-I)

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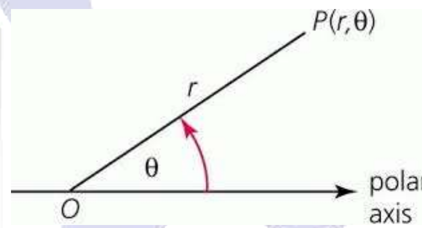
UNIT-5: STRAIGHT LINES

Topic 5.1: Cartesian and Polar Coordinate:

- In the Cartesian plane, the horizontal line is called the *x*-axis and the vertical line is called the *y*-axis.
- The coordinate axes divide the plane into four parts called quadrants.
- The point of intersection of the axes is called the origin.
- Abscissa or the *x*-coordinate of a point is its distance from the y-axis and the ordinate or the *y*-coordinate is its distance from the *x*-axis,
- (x, y) are called the coordinates of the point whose abscissa is x and the ordinate is y,
- Coordinates of a point on the *x*-axis are of the form $(x, 0)$ and that of the point on the *y*-axis is of the form $(0, y)$.
- The coordinates of the origin are (0, 0).
- Signs of the coordinates of a point in the first quadrant are (+, +), in the second quadrant (–, +), in the third quadrant (–, –) and in the fourth quadrant (+, –).

Polar Co-ordinates

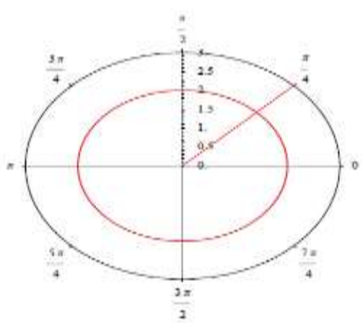
A polar coordinate system, gives the co-ordinates of a point with reference to a point O and a half line or ray starting at the point O. We will look at polar coordinates for points in the *xy*-plane, using the origin (0, 0) and the positive *x*-axis for reference. A point P in the plane, has polar coordinate (r, θ), where r is the distance of the point from the origin and θ is the angle that the ray $|OP|$ makes with the positive *x*-axis.



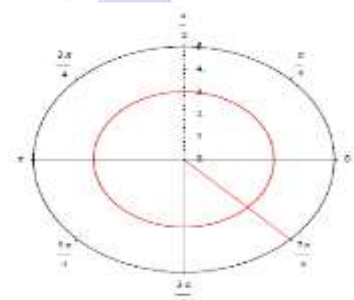
Example: Plot the points whose polar coordinates are given by $\left(2, \frac{\pi}{4}\right)$, $\left(3, \frac{7\pi}{4}\right)$

A: $\left(2, \frac{\pi}{4}\right)$

$\left(3, \frac{7\pi}{4}\right)$



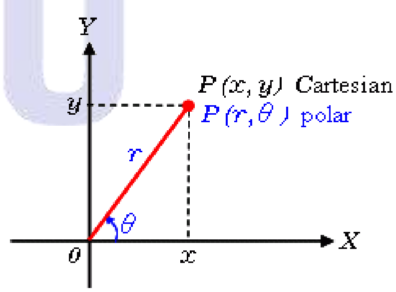
B:



- The representation of a point in polar coordinate is not unique.

Polar to Cartesian coordinates and Vice-Versa Conversion:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\text{where, } r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



Example: To convert $(1, 1)$ to polar coordinates.

Answer: Here,

$$x = 1, \quad y = 1$$

Formula: $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

So the required polar coordinate = $(r, \theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$

Example: To convert $\left(3, -\frac{\pi}{3}\right)$ in Cartesian coordinates.

Answer: Here,

$$r = 3, \quad \theta = -\frac{\pi}{3}$$

Formula: $x = r\cos\theta, \qquad y = r\sin\theta$

$$x = 3\cos\left(-\frac{\pi}{3}\right), y = 3\sin\left(-\frac{\pi}{3}\right)$$

$$x = 3 \times \left(\frac{1}{2}\right), \qquad y = 3\left(-\frac{\sqrt{3}}{2}\right)$$

$$x = \frac{3}{2}, \quad y = -\frac{3\sqrt{3}}{2}$$

So the required Cartesian coordinate = $(x, y) = \left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$

- Distance between the points P (x_1, y_1) and Q (x_2, y_2):

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Distance between $(4,6)$ and $(0,3)$ is $= \sqrt{(0-4)^2 + (3-6)^2} = \sqrt{16+9} = 5$ units

- The coordinates of the mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Area of triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is:

$$A = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Example: The area of the triangle, whose vertices are $(0,0),(1,0)$ and $(0,1)$ is

$$A=\frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$$

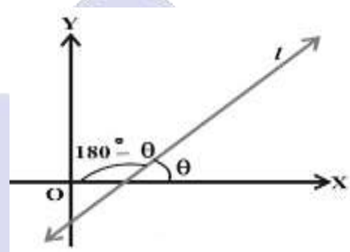
$$A=\frac{1}{2}|0(0-1)+1(1-0)+0(0-0)|$$

$$A=\frac{1}{2}\text{ units}$$

- If the area of the triangle *ABC* is zero, then three points *A*, *B* and *C* lie on a line, *i.e.* they are collinear.

Slope of a line: If θ is the inclination of a line, then $\tan\theta$ is called the *slope or gradient* of the line. The slope of a line whose inclination is 90^0 is not defined. The slope of a line is denoted by *m*. Thus $m=\tan\theta,\ \theta\neq 90^0$.

It may be observed that the slope of x-axis is zero and slope of y-axis is not defined.



- **Slope of line passing through** $(x_1,y_1),(x_2,y_2)$**:**

Formula: $m=\frac{y_2-y_1}{x_2-x_1}$

Example: To find the slope of line passing through points $(3,-2)$ and $(7,-2)$

Answer: Here, $(x_1,y_1)=(3,-2),\ (x_2,y_2)=(7,-2)$

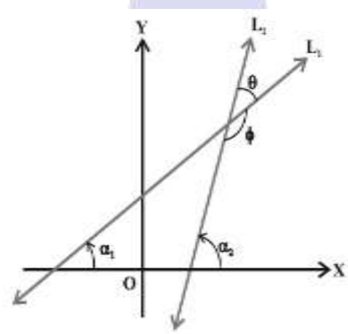
Formula $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-(-2)}{7-3}=0$

- **Angle between two lines:** Let L_1 and L_2 be two non-vertical lines with slope m_1 and m_2 respectively. Let α_1 and α_2 are the inclination of lines L_1 and L_2.

Then $m_1=\tan\alpha_1,\ m_2=\tan\alpha_2$

Let θ be the angle of intersection then,

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$$



Note: 1. Two lines are parallel if $m_1=m_2$

2. Two lines are perpendicular if $m_1m_2=-1$

Example: If the angle between two lines is $\frac{\pi}{4}$ and slope of the lines is $\frac{1}{2}$, find the slope of the other line.

Answer: Formula: $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

Given $m_1 = \frac{1}{2}$, $m_2 = m$ and $\theta = \frac{\pi}{4}$

Therefore,

$$\tan\frac{\pi}{4} = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right| \quad \Rightarrow 1 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|$$

$$\Rightarrow 1 = \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} \quad \text{or} \quad \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = -1$$

$$\Rightarrow m = 3 \quad \text{or} \quad m = -\frac{1}{3}$$

Topic 5.2: Different Forms of a Straight Line:

- **Point Slope Form:** Equation of line passing through point (x_0, y_0) and slope m is given by:

$$y - y_0 = m(x - x_0)$$

Example: To find the equation of line through point $(-2,3)$ with slope -4

Answer: Here, $(x_0, y_0) = (-2,3)$ and $m = -4$

Formula: $y - y_0 = m(x - x_0)$

$$\Rightarrow y - 3 = -4(x + 2) \quad \Rightarrow 4x + y + 5 = 0$$

- **Two-Point Form:** Equation of line passing through points (x_1, y_1) and (x_2, y_2) is given by:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Example: To find the equation of line passing through the points $(1,-1)$ and $(3,5)$

Answer: Here, $(x_1, y_1) = (1,-1)$ and $(x_2, y_2) = (3,5)$

Formula: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$\Rightarrow y-(-1)=\frac{5-(-1)}{3-1}(x-1)$$
$$\Rightarrow -3x+y+4=0$$

- **<u>Slope- Intercept Form:</u>** Equation of line with slope m and intercept c on *y*-axis is given by:

$$y=mx+c$$

<u>Example:</u> To find the equation of line with slope -1 and intercept of 2 on *y*-axis.

<u>Answer:</u> Here, $m=-1$ and $c=2$

Formula: $y=mx+c$

$$\Rightarrow y=-x+2 \quad \Rightarrow x+y-2=0$$

- **<u>Intercept Form:</u>** Equation of line making intercepts of a and b on *x*-axis and *y*-axis respectively is given by: $$\frac{x}{a}+\frac{y}{b}=1$$

<u>Example:</u> To find the equation of line, which makes intercepts -2 and 3 on *x*-axis and *y*-axis respectively.

<u>Answer:</u> Here, $a=-2$ and b = 3

Formula: $\frac{x}{a}+\frac{y}{b}=1$

$$\Rightarrow \frac{x}{-2}+\frac{y}{3}=1$$
$$\Rightarrow 3x-2y+6=0$$

<u>Topic 3.3: General Equation of a Line:</u>

General equation of line is given by: $Ax+By+C=0$

Where A, B are not zero. The slope of equation is $m=-\frac{A}{B}$

<u>Example:</u> Equation of line is $3x-4y+12=0$. To find its (I) Slope and (II) *x*- and *y*-intercepts.

<u>Answer:</u> (I) Given equation of line: $3x-4y+12=0$

$$\Rightarrow 4y=3x+12$$
$$\Rightarrow y=\left(\frac{3}{4}\right)x+3$$

Comparing with slope-intercept form $y=mx+c$, we have slope of the line $m=\frac{3}{4}$

(II) Given equation of line: $3x-4y+12=0$

$$\Rightarrow 3x-4y=-12$$

$$\Rightarrow \left(\frac{3x}{-12}\right)+\left(\frac{-4y}{-12}\right)=0$$

$$\Rightarrow \frac{x}{\left(\frac{-12}{3}\right)}+\frac{y}{\left(\frac{-12}{-4}\right)}=0 \Rightarrow \frac{x}{(-4)}+\frac{y}{(3)}=0$$

Comparing with intercepts form of line $\frac{x}{a}+\frac{y}{b}=1$, we have $a=-4,\ b=3$

- **<u>Topic 3.4: Distance of a Point from a Line:</u>** The distance of a point from a given line is the length of the perpendicular drawn from the point to the line. Let equation of line is: $Ax+By+C=0$. Its distance from the point $P(x_1, y_1)$ is d .

$$\text{Then} \quad d=\frac{|Ax_1+By_1+C|}{\sqrt{x^2+y^2}}$$

<u>Example:</u> To find the distance of the point $(1,-2)$ from the line $4x-3y+9=0$.

<u>Answer:</u> Given point $(x_1, y_1)=(1,-2)$ and line $4x-3y+9=0$

Comparing with general equation of line $Ax+By+C=0$, we have: $A=4, B=-3, C=9$

$$\text{Formula: } d=\frac{|Ax_1+By_1+C|}{\sqrt{x^2+y^2}}$$

$$\Rightarrow d=\frac{|4\times 1-3\times(-2)+9|}{\sqrt{4^2+(-3)^2}}$$

$$\Rightarrow d=\frac{|4+6+9|}{\sqrt{16+9}} \quad \Rightarrow d=\frac{15}{5} \quad \Rightarrow d=3\text{units}$$